Econometric Analysis of Games 1

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HT 2017
Recap

**Aim:** provide an introduction to incomplete models and partial identification in the context of discrete games

1. Coherence & Completeness
2. Basic Framework
3. Pooling Outcomes
4. Large Support
5. Partial Identification
6. Adding Structure
Recap

► Last lecture discussed the rank and order conditions that characterise when linear simultaneous systems are point identified.

► This week, we will review identification results in the econometric analysis of discrete games, which are also simultaneous systems

► The existence of multiple equilibria results in additional identification issues

► After discussing games of complete information, we will return to multiplicity in the context of social interactions
Completeness & Coherence

- Structural model: a set of structural equations and a distribution between observed and unobserved explanatory variables specified by an economic theory

\[ h(y, X, \varepsilon) = 0 \]  

with \( \varepsilon \in \Omega \)

- The model is coherent if for each \( \varepsilon \in \Omega \) there exists at least one value of \( y \) that satisfies the structural equations

- The model is complete if for each \( \varepsilon \in \Omega \) there exists at most one value of \( y \) that satisfies the structural equations

- With a complete and coherent model, a unique reduced form is guaranteed
Completeness & Coherence

- Incoherent and incomplete models can arise in simultaneous systems
- We start with a simple abstract example, and then will consider these issues in the context of an entry game
- Consider the following simultaneous system

\[
\begin{align*}
y_1 &= l(y_2 + \epsilon_1 \geq 0) \\
y_2 &= \theta y_1 + \epsilon_2
\end{align*}
\]
We have that

$$y_1 = l(\theta y_1 + \epsilon_1 + \epsilon_2 \geq 0)$$  \hspace{1cm} (3)$$

With unrestricted errors, the model is incomplete: let $-\theta \leq \epsilon_1 + \epsilon_2 < 0$:

$$y_1 = l(\theta y_1 + \epsilon_1 + \epsilon_2 \geq 0)$$

$$\epsilon_1 + \epsilon_2 \geq 0 \rightarrow y_1 = 0$$

$$\theta + \epsilon_1 + \epsilon_2 \geq 0 \rightarrow y_1 = 1$$  \hspace{1cm} (4)$$
With unrestricted errors, the model is incoherent: let $0 \leq \epsilon_1 + \epsilon_2 < -\theta$:

\[
y_1 = I(\theta y_1 + \epsilon_1 + \epsilon_2 \geq 0)
\]

\[
\epsilon_1 + \epsilon_2 \geq 0 \rightarrow y_1 \neq 0
\]

\[
\theta + \epsilon_1 + \epsilon_2 < 0 \rightarrow y_1 \neq 1
\]
Outline

1. Coherence & Completeness
2. Basic Framework
3. Pooling Outcomes
4. Large Support
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Entry Games
Entry Games

- Large literature on the estimation on binary games in the empirical IO set-up
- How to measure the competitiveness of a market and the toughness of competition?
- Bresnahan & Reiss (1991): don’t need to price/quantity/cost data but can infer from information on equilibrium market structure
Entry Games: Data

- A firm’s decision to enter depends on its profit, which depends on whether the other firm entered the market.

- Let profits be given by:

$$\pi_i = x_i \beta + \Delta_i y_j + \epsilon_i$$

for $i = 1, 2$

- The distribution of $\epsilon \equiv (\epsilon_1, \epsilon_2)$ is given by (known) $F$, with mean normalised to zero and variances to 1.

- Start by assuming complete information: realisations of $x$ and $\epsilon$ observed by all players.
Entry Games: Data

- Similar set-up to Ciliberto & Tamer’s (2009) analysis of the airline industry
- Each market is a particular route, the firms being different airlines
- $x_i$ gives the various market and firm specific variables that affect demand (e.g. population size, income &c) and costs for firms
- Linearity relaxed in some specifications, but additive separability of observables and unobservables typically assumed
Payoff Matrix

\[
y_i^* = x_i \beta + \Delta_i y_j + \epsilon_i \\
y_i = I(y_i^* \geq 0)
\]  

<table>
<thead>
<tr>
<th></th>
<th>Player 1</th>
<th>Player 2</th>
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<tbody>
<tr>
<td>0</td>
<td>0, 0</td>
<td>0, (x_2 \beta_2 + \epsilon_2)</td>
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<td>1</td>
<td>(x_1 \beta_1 + \epsilon_1, 0)</td>
<td>(x_1 \beta_1 + \Delta_1 + \epsilon_1, x_2 \beta_2 + \Delta_2 + \epsilon_2)</td>
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- Nash equilibrium concept
Payoff Matrix: $\Delta_i < 0$
Incomplete

- For certain values of the error term, the model predicts two possible solutions.

- Absent a rule for equilibrium selection, the model is incomplete — cannot write down uniquely determined likelihood function because we cannot assign separate probabilities to (0,1) and (1,0).

- Structural parameters often under-identified from entry data.

- An equilibrium selection mechanism, $\lambda$, specifies which equilibria is selected with which probability.
Incomplete

- How can we deal with this?
  - Find a coarser unique prediction — e.g. focus on number of entrants — useful in a symmetric world but doesn’t always work (Bresnahan and Reiss, 1991; Berry, 1992)
  - Exclusion & large support assumptions (Tamer, 2003)
  - Complete the model by assuming an equilibrium selection rule or impose more structure on the game to obtain a unique equilibrium
  - Sacrifice point estimates and only find bounds on parameters
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Pooling Multiple Equilibria Outcomes

- A common strategy taken is to aggregate the monopoly outcomes
- Transform model from one that explains each entrants strategy to one that predicts the number of entrants: $N = 0, 1, 2$
- Essentially turns the problem into an ordered discrete choice model
- Multiplicity not necessarily an impediment to analysis — here some quantities of interest are invariant across equilibria
Pooling Multiple Equilibria Outcomes

- Likelihood function would then include the probability statements

\[
Pr(N = 0 | x) = Pr(x_1 \beta_1 + \epsilon_1 < 0, x_2 \beta_2 + \epsilon_2) = F_{\epsilon_1, \epsilon_2}(-x_1 \beta_1, -x_2 \beta_2)
\]

\[
Pr(N = 2 | x) = Pr(x_1 \beta_1 + \Delta_1 + \epsilon_1 < 0, x_2 \beta_2 + \Delta_2 + \epsilon_2) = F_{\epsilon_1, \epsilon_2}(-x_1 \beta_1, -x_2 \beta_2)
\]

\[
Pr(N = 1 | x) = 1 - Pr(N = 0 | x) - Pr(N = 2 | x)
\]

(8)

- Joint distribution of \( \Delta_i \) determines the specific functional form for these probability statements
Pooling Multiple Equilibria Outcomes

- However, lose information about firm heterogeneities unless firms are symmetric

- We then have

\[
\begin{align*}
Pr(N = 0|x) &= Pr(\epsilon_1 < -x\beta) \\
Pr(N = 1|x) &= Pr(-x\beta < \epsilon < -x\beta - \Delta) \\
Pr(N = 2|x) &= Pr(-x\beta - \Delta < \epsilon)
\end{align*}
\]  

- With \( \epsilon \sim N(0, 1) \) we get a very simple ordered probit model
Pooling Multiple Equilibria Outcomes

- While convenient, this strategy imposes strong restrictions on firm level heterogeneity
- Typically want to allow for firms to asymmetric impacts on one another
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Large Support

- Uses similar ideas to those we have discussed in the context of special regressors and identification of simultaneous systems

- Intuition: find a regressor for which the actions of all but one player are dictated by dominant strategies — turn the problem into a discrete choice problem by the single agent who does not play dominant strategies

- For $i = 1$ or $i = 2$, there exists a regressor with $\beta_{ik} \neq 0$ that is excluded from the other firm’s covariates and has everywhere positive density
Large Support

- Point identification of \((\beta_1, \beta_2, \Delta_1, \Delta_2)\) then guaranteed if \(x_1\) and \(x_2\) have full column rank

- Can find extreme values for \(x_{ik}\) such that only unique equilibria in pure strategies are realised, e.g. as \(x_{1k} \to -\infty\):

\[
Pr(\epsilon_1 < -x_1 \beta) \to 1
\]

\[
Pr(\epsilon_1 < -x_1 - \Delta_1 \beta) \to 1
\]

- Let \(x_2^*\) be such that:

\[
x_2^* \beta_2 \neq x_2^* b_2
\]

which is guaranteed by the full rank condition on \(x_2\)
We then have that, as $x_{1k} \to \infty$

$$Pr((0, 0)|x) = Pr(\epsilon_1 < -x_1 \beta_1, \epsilon_2 < -x_2^* \beta_2)$$
$$\approx Pr(\epsilon_2 < -x_2^* \beta_2)$$
$$\not\approx Pr(\epsilon_2 < -x_2^* b_2)$$

which implies that $\beta_2$ is identified
Large Support

▶ Let $x_1^*$ be such that:

$$x_1^* \beta_1 \neq x_1^* b_1 \quad (13)$$

which is guaranteed by the full rank condition on $x_1$

▶ We then have that

$$Pr((0, 0) | x) = Pr(\epsilon_1 < -x_1^* \beta_1, \epsilon_2 < -x_2 \beta_2) \neq Pr(\epsilon_1 < -x_1^* b_1, \epsilon_2 < -x_2 \beta_2) \quad (14)$$

which implies that $\beta_1$ is identified

▶ Could also introduce a separate large support regressor for firm 2
Identification at Infinity

- This strategy is often called **identification at infinity** — using independent variation in one regressor while driving another to take extreme values on its support identifies the parameters.

- Used in many different applications — Heckman (1990) &c

- However, note that there are important implications for inference: this will lead to asymptotic convergence rates that are slower than parametric rates as the sample size (i.e. number of games) increases (see Kahn & Tamer (2010); Bajari et al (2011))
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Partial Identification

- Typically covariates’ support is not rich enough to admit a strategy based on Tamer (2003)
- Can instead rely on partial information and use bounds for estimation — while a model may not make exact predictions, it may still meaningfully restrict the range of possible outcomes
- Advocated by, e.g., Manski (1995)
- These results can be useful for testing particular hypotheses or illustrating the range of possible outcomes
Partial Identification

- Given that the model is incomplete, the probability of outcome \((0, 1)\) and \((1, 0)\) cannot be written as a function of the structural parameters

- The model instead provides upper and lower bounds on these probabilities
Bounds

\[ Pr((0, 1)|x) \geq P_{L,01}(\beta, \triangle) \]

\[ = Pr(\epsilon_1 < -x_1 \beta_1, -x_2 \beta_2 < \epsilon_2) \]

\[ + Pr(-x_1 \beta_1 \leq \epsilon_1 < -x_1 \beta_1 - \triangle_1, -x_2 \beta_2 - \triangle_2 < \epsilon_2) \]

(15)
Bounds

\[ Pr((0, 1) \mid x) \geq P_{L,01}(\beta, \Delta) \]

\[ = Pr(\epsilon_1 < -x_1\beta_1, -x_2\beta_2 < \epsilon_2) \]

\[ + Pr(-x_1\beta_1 \leq \epsilon_1 < -x_1\beta_1 - \Delta_1, -x_2\beta_2 - \Delta_2 < \epsilon_2) \]

(16)
Bounds

\[ Pr((0, 1)|x) \geq P_{L,01}(\beta, \Delta) \]

\[ = Pr(\epsilon_1 < -x_1\beta_1, -x_2\beta_2 < \epsilon_2) \]

\[ + Pr(-x_1\beta_1 \leq \epsilon_1 < -x_1\beta_1 - \Delta_1, -x_2\beta_2 - \Delta_2 < \epsilon_2) \]

(17)
**Bounds**

\[
Pr((0, 1) | x) \geq P_{L,01}(\beta, \triangle) \\
= Pr(\epsilon_1 < -x_1 \beta_1, -x_2 \beta_2 < \epsilon_2) \\
+ Pr(-x_1 \beta_1 \leq \epsilon_1 < -x_1 \beta_1 - \triangle_1, -x_2 \beta_2 - \triangle_2 < \epsilon_2)
\]  

(18)
Bounds

\[ Pr((0, 1) \mid x) \geq P_{L,01}(\beta, \triangle) \]

\[ = Pr(\epsilon_1 < -x_1 \beta_1, -x_2 \beta_2 < \epsilon_2) \]

\[ + Pr(-x_1 \beta_1 \leq \epsilon_1 < -x_1 \beta_1 - \triangle_1, -x_2 \beta_2 - \triangle_2 < \epsilon_2) \]

(19)
Bounds

\[ Pr((0, 1)|x) \leq P_{U,01}(\beta, \Delta) \]

\[ = Pr \left( \begin{array}{l}
-x_1\beta_1 \leq \epsilon_1 < -x_1\beta_1 - \Delta_1, \\
-x_2\beta_2 < \epsilon_2 < -x_2\beta_2 - \Delta_2
\end{array} \right) + P_{L,01} \]

(20)
Bounds

\[
Pr((0, 1)|x) \leq P_{U,01}(\beta, \Delta) = Pr \left( \begin{array}{l}
-x_1/\beta_1 \leq \epsilon_1 < -x_1/\beta_1 - \Delta_1, \\
-x_2/\beta_2 < \epsilon_2 < -x_2/\beta_2 - \Delta_2,
\end{array} \right) + P_{L,01}
\]
Partial Identification

- Can derive similar expressions for the probabilities of other outcomes

- The information about the parameters can then be represented as:

\[
\begin{pmatrix}
P_{L,00}(\theta) \\
P_{L,01}(\theta) \\
P_{L,10}(\theta) \\
P_{L,11}(\theta)
\end{pmatrix} \leq \begin{pmatrix}
Pr((0, 0)|x) \\
Pr((0, 1)|x) \\
Pr((1, 0)|x) \\
Pr((1, 1)|x)
\end{pmatrix} \leq \begin{pmatrix}
P_{U,00}(\theta) \\
P_{U,01}(\theta) \\
P_{U,10}(\theta) \\
P_{U,11}(\theta)
\end{pmatrix}
\]

- The identified set, \( \Theta_I \), is the set of all parameters that satisfy these inequalities
Example: Ciliberto & Tamer (2009)

- Interested in measuring competitive pressures in the airline industry, i.e. the impact on profit of a firm entering a particular market.

- Want to allow for heterogeneity across firms — i.e. the entry decision of American Airlines has a different effect on the profit of its competitors than the entry of a low cost airline.

- Some important policy questions regards rules on airport presence on number of routes served &c.
Example: Ciliberto & Tamer (2009)

- Each observation is a route served between two airports.
- Cross section study and thus assume firms are in long run equilibrium.
- Firm profit function given by:

\[
y_{im}^* = S_m \alpha_i + Z_{im} \beta_i + W_{im} \gamma_i + \sum_{j \neq i} \theta_j^i y_{jm} + \sum_{j \neq i} Z_{jm} \theta_j^i y_{jm} + \epsilon_{im}
\]

(23)

- $S$ is a vector of common market characteristics.
- $Z$ is a matrix of firm characteristics that enter into the profit functions of all firms.
- $W$ are firm specific characteristics such as cost variables.
Example: Ciliberto & Tamer (2009)

- Have the (more complicated!) set of moment equalities

\[
\begin{pmatrix}
P_{L,1}(\theta) \\
\vdots \\
P_{L,2^N}(\theta)
\end{pmatrix}
\leq
\begin{pmatrix}
Pr(y^1|x) \\
\vdots \\
Pr(y^{2^N}|x)
\end{pmatrix}
\leq
\begin{pmatrix}
P_{U,1}(\theta) \\
\vdots \\
P_{U,2^N}(\theta)
\end{pmatrix}
\]

(24)
Example: Ciliberto & Tamer (2009)

The estimator uses the following objective function

\[
Q(\theta) = \int \left[ \| (\Pr(y|X) - P_L(X, \theta))_\cdot \| + \| (\Pr(y|X) - P_H(X, \theta))_+ \| \right] dF_x
\]

where

\[
(A)_- = \begin{bmatrix}
a_11(a_1 \leq 0) \\
\vdots \\
a_{2N}1(a_{2N} \leq 0)
\end{bmatrix}
\]

and \((A_+)\) defined similarly.
Example: Ciliberto & Tamer (2009)

- Note that $Q(\theta) \geq 0$ and $Q(\theta) = 0$ iff $\theta \in \Theta_I$

- To estimate $\Theta_I$, take the sample analog of $Q(\cdot)$

$$Q_N(\theta) = \frac{1}{N} \sum_{m} \left[ || (P_n(X_m) - P_L(X_m, \theta))_+ || + || (P_n(X_m) - P_H(X_m, \theta))_+ || \right]$$

(27)

where $P_n(X_m)$ can be estimated non-parametrically and $P_L$ and $P_H$ are computed via simulation.
Example: Ciliberto & Tamer (2009)

- Unless the number of firms is very small, the upper and lower bound probabilities do not have a convenient closed form.

- Common problem with more complicated models: likelihood or moment inequalities do not have a closed form that can adapt standard Maximum Likelihood or GMM methods for.

- Proceed by **simulation**: calculate an upper and lower bound for each equilibrium probability for every $X$ for a particular guess of the parameter values.
Simulation Procedure

1. Draw $R$ simulations of the firm unobservables, $\epsilon^r$ — these draws and stored and held fixed throughout the optimisation procedure
   - NB can allow for correlation in these draws if each firm’s draw is random normal by transforming the error terms using the Cholesky Decomposition of the covariance matrix

2. For a given $X$, a particular draw of the error term, $\epsilon^r$ and an initial guess of the parameter vector, $\theta$, calculate the vector of firm’s profits for a particular set of entry decisions, $y^j$

3. If each firm is earning non-negative profits, then the outcome $y^j$ is an equilibrium
Simulation Procedure

4. If this equilibrium is unique (for no other $y'$ is it true that all firms profits are positive), then add $\frac{1}{R}$ to the lower and upper bound for this outcome.

5. If this equilibrium is not unique, then add $\frac{1}{R}$ to the upper bound only.

6. Repeat steps 1-5 for each $X_m$ and $\epsilon^r$, $r = 1, \ldots, R$.

7. Calculate the objective simulation estimates of $P_L$ and $P_H$.

8. Repeat steps 1-7 as search for the value of $\theta$ that minimises $Q_N$.  

Abi Adams

TBEA
Data

- 2001 Airline Origin and Destination Survey
- Markets defined as trips between airports — data set includes 2742 markets
- Focus on American, Delta, United, Southwest, ‘Medium’ Airlines and Low Cost Carriers
Results Summary

- Heterogeneity in profit functions
- Competitive effects of large and low-cost airlines are different
- Competitive effects of an airline increasing in its airport presence
### Results

**EMPIRICAL RESULTS**

<table>
<thead>
<tr>
<th></th>
<th>Berry (1992)</th>
<th>Heterogeneous Interaction</th>
<th>Heterogeneous Control</th>
<th>Firm-to-Firm Interaction</th>
</tr>
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<tbody>
<tr>
<td>Competitive fixed effect</td>
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<td>Market size</td>
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<td>LCC</td>
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(Continues)
## Results

<table>
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<th>Complete Entry Pooling Large Support Partial</th>
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### TABLE III—Continued

<table>
<thead>
<tr>
<th></th>
<th>Berry (1992)</th>
<th>Heterogeneous Interaction</th>
<th>Heterogeneous Control</th>
<th>Firm-to-Firm Interaction</th>
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<tbody>
<tr>
<td>Market distance</td>
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<td>Per capita income</td>
<td>[0.568, 2.623]</td>
<td>[-0.080, 1.010]</td>
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<td>[0.272, 1.073]</td>
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<td>Income growth rate</td>
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## Results

### Variable Competitive Effects

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<th>Independent Unobs</th>
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<th>Only Costs</th>
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<td><strong>Fixed effect</strong></td>
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<tr>
<td>UA</td>
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<td>[−4.580, −3.813]</td>
<td>[−10.671, −8.386]</td>
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<td><strong>Variable effect</strong></td>
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<td><strong>Cost</strong></td>
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## Results

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<th>Pooling</th>
<th>Large Support</th>
<th>Partial</th>
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<td>[1.946, 2.435]</td>
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<td>[0.260, 0.612]</td>
<td>[0.823, 1.068]</td>
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<td>Market distance</td>
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</table>
Conclusion

- To finish, note that many other economic models with similar structures

- (Especially!) with strategic interactions, need to consider if your model is complete and consider the implications for identification

- Identification can be achieved through a number of strategies; which one is preferable will depend on the context and data to hand